

Supplementary Information ' Sensitivity of dynamical systems instantaneous dimension and its insights on sea surface temperature anomaly field over the tropical Pacific '

Ming Shi¹, Huang Yu^{2,3}, Zuntao Fu^{4*}, Hao Xue¹, Xiaolong Qiu¹, Xintong Hao¹, Xiaohan Zhang⁵, Haiyang Yu⁵, Weiqi Xing⁶

¹Renewable Energy Survey and Design Department, Huaneng Clean Energy Research Institute, Beijing, 102209, China

²Munich Climate Center and Earth System Modelling Group, Department of Aerospace and Geodesy, TUM School of Engineering and Design, Technical University of Munich, 80333, Germany.

³Complexity Science, Potsdam Institute for Climate Impact Research, Potsdam, 14412, Germany

⁴Laboratory for Climate and Ocean-Atmosphere Studies, Department of Atmospheric and Oceanic Sciences, School of Physics, Peking University, Beijing, 100871, China

⁵Engineering Department, China Huaneng Group Corporation Limited, 070001, China

⁶Qinghai Branch of China Huaneng Group Corporation Limited, 810008, China

The present Supporting Information includes the sensitivity analysis with respect to the choice of quantile thresholds (5% and 1%).

Previous studies have shown that the performance of the dynamical systems (DS) method is not sensitive to the choice of the threshold. In the literature, Faranda et al.(2017) applied the DS method to investigate the predictability and extremes of atmospheric flow over the North Atlantic[1].They initially set the threshold p to 0.98 (i.e., the top 2% of values), and subsequently examined the robustness of the results under various threshold configurations. It was found that the results remained stable when the threshold p ranged from 0.975 to 0.99. In fact, if too high a threshold is selected, the number of values exceeding it is insufficient to successfully fit the generalized Pareto distribution. Datsiris et al.(2023) indicated that the choice of quantile threshold p should satisfy the condition that $N(1 - p) \geq 100-1000$ (N is the data length), to ensure that the number of exceedances is sufficient for a reliable fit[2]. And the quantile threshold p should satisfy the NRMSE test[2]. In this study, the quantile threshold $p=0.95$, 0.98, and 0.99 all satisfy this requirement.

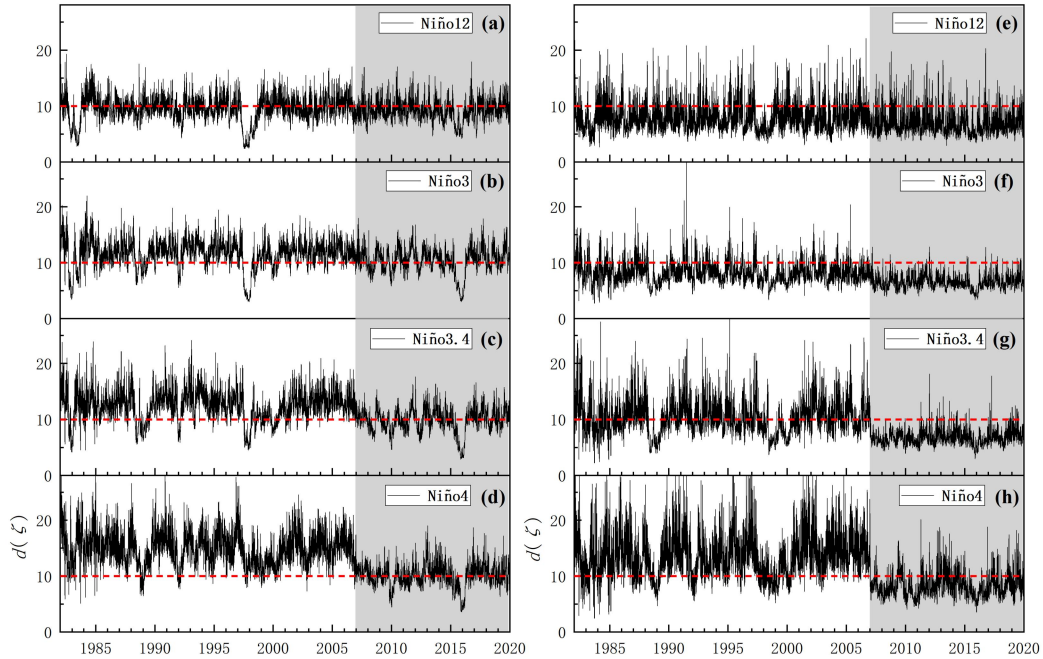


Figure R1. Evolution of the instantaneous dimension d of SST at different thresholds over four typical Niño regions. The threshold is 5% quantile for (a)-(d) and 1% quantile for (e)-(h). Panels (a) and (e) correspond to Niño1+2 area, (b) and (f) to Niño3 area, (c) and (g) to Niño3.4 area, and (d) and (h) to Niño4 area. The gray shaded area represents the years after 2007. Red dashed lines for the eye-guided variation of DSID mean value.

To assess the robustness of our results, we performed a sensitivity analysis with respect to the threshold choice. The 5% ($p = 0.95$) and 1% ($p = 0.99$) quantiles were adopted as thresholds for identifying analogous states, and the DSID of SST was computed for the four Niño areas. The results are presented in Fig. R1, Table R1, and Table R2. When the stricter threshold (1%) is applied, only the nearest analogous states in the phase space are selected for each reference point ξ . As a result, the chosen phase-space trajectories become more convergent, leading to a lower mean of

the computed DSID series (see \bar{d} in Tables R1 and R2). Nonetheless, although the DSID series obtained with the two thresholds (5% and 1%) exhibit minor differences, both series clearly capture a significant mean shift around 2007. This confirms that the choice of threshold does not compromise the ability of the DSID to detect changes in dynamical states. We therefore conclude that the selection of the quantile threshold does not affect the findings of this study.

Table R1. Statistics of the mean and variance of the DSID before and after 2007 at the 5% quantile threshold. Bold for significant at the level of 0.001 from moving block bootstrap (MBB) test.

| | \bar{d} | | | σ^2 | | |
|-----------------|-------------|------------|-------------------------|-------------|------------|--------------------------|
| | Before 2007 | After 2007 | \bar{d}_{Diff} | Before 2007 | After 2007 | σ^2_{Diff} |
| Niño 1+2 | 9.41 | 9.07 | -0.34 | 5.09 | 3.39 | -1.70 |
| Niño 3 | 11.49 | 10.30 | -1.19 | 6.15 | 4.89 | -1.25 |
| Niño 3.4 | 12.30 | 9.73 | -2.58 | 7.40 | 4.67 | -2.73 |
| Niño 4 | 14.73 | 10.00 | -4.72 | 8.90 | 4.13 | -4.77 |

Table R2. Statistics of the mean and variance of the DSID before and after 2007 at the 1% quantile threshold. Bold for significant at the level of 0.001 from moving block bootstrap (MBB) test.

| | \bar{d} | | | σ^2 | | |
|-----------------|-------------|------------|-------------------------|-------------|------------|--------------------------|
| | Before 2007 | After 2007 | \bar{d}_{Diff} | Before 2007 | After 2007 | σ^2_{Diff} |
| Niño 1+2 | 7.75 | 6.77 | -0.99 | 5.68 | 3.77 | -1.90 |
| Niño 3 | 8.12 | 6.49 | -1.63 | 3.59 | 1.41 | -2.18 |
| Niño 3.4 | 10.11 | 6.70 | -3.40 | 8.43 | 1.74 | -6.69 |
| Niño 4 | 13.35 | 8.12 | -5.23 | 14.60 | 3.92 | -10.68 |

Reference

- [1] Faranda, D., Messori, G., and Yiou, P.: Dynamical proxies of North Atlantic predictability and extremes, *Sci Rep*, 7, 41278, <https://doi.org/10.1038/srep41278>, 2017.
- [2] Datsleris, G., Kottlarz, I., Braun, A. P., and Parlitz, U.: Estimating fractal dimensions: A comparative review and open source implementations, *Chaos: An Interdisciplinary Journal of Nonlinear Science*, 33(10), <https://doi.org/10.1063/5.0160394>, 2023.